THE SYNTHESIS ALGORITHMS OF OPTIMAL REGULATORS FOR DISCRETE SYSTEMS BASED ON REGULATION QUALITY CRITERION

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Abstract

Algorithms for the synthesis of optimal controllers for discrete systems based on the control quality criterion are presented. When constructing the transfer function of the optimal controller in order to simplify the computing system, the polynomials of the indicated transfer function were used. An improved method is proposed for constructing optimal controllers that provide a minimum of the root-mean-square performance criterion for discrete-continuous systems, which does not require a laborious auxiliary procedure for solving a system of polynomial equations as one of the stages of constructing a controller. The given computational schemes made it possible to solve the problem of synthesis of optimal controllers in control systems of industrial technological processes effectively.

Keywords: discrete-continuous systems, optimal controller, synthesis algorithms, root-mean-square function.

Introduction

One of the most important issues to be solved in the practical implementation of control laws, including optimal ones, is the choice of the structure of the realizing technical device, i.e. the choice of the composition of individual elements and the provision of appropriate links between them [1-3]. At the same time, it is obvious that the structure of the device being implemented is largely related to the structure of the control law. Systems for automatic control of complex objects often contain various digital devices as components. Therefore, it is necessary to adapt existing or develop new mathematical methods for finding optimal controllers in relation to the technical means on which they are implemented [1-6]. Nevertheless, the study of the structural properties of optimal control laws seems to be a task that deserves considerable attention. In this case, it is an interest to assess the structural properties of the controller from the initial data, without solving the synthesis problem firstly. We mean the set of degrees of the numerator and denominator of its transfer function by the structure of the controller.

Formulation of the problem

Let us assume that the controlled object is described by the equation:

$$A(q)x_t = H(q)u_t + \varphi_t, \tag{1}$$

where x_t, u_t –the object coordinate and control at the moments of time t simultaneously; φ_t – external disturbance with known correlation coefficients $\varepsilon(i) = \varepsilon(-i) = <\varphi_t \varphi_{t+i}>$; $H(q) = h_0 q^k + h_1 q^{k+1} + \ldots + h_l q^{k+l}$; $A(q) = a_0 + a_1 q + \ldots + a_{n-1} q^{n-1} + a_n q^n$, $(a_0 \neq 0)$ – polynomials with constant coefficients; q – operator of shift backward.

Along with system (1), consider the mean-square function (control quality criterion) [1,3]:

$$I = \lambda < x_t^2 > + \mu < \mu_t^2 > = \overline{\lim_{t \to \infty}} M\left\{\lambda x_t^2 + \mu u_t^2\right\}, \tag{2}$$

where are the factors $\lambda \geq 0,\ \mu \geq 0,\ \lambda + \mu \neq 0$; $M\{\cdot\}$ – expectation sign.

We will assume that the control u_t for object (1) is formed in the form of a controller (feedback) described by the equation:

$$u_t = W(q)x_t, \tag{3}$$

where $W(q) = W_1(q)/W_2(q)$ – regulator transfer function, $W_1(q)$ и $W_2(q)$ – polynomials.

Applicable z-transformation to the equations of the object (1) and the controller (3)

$$\begin{gathered} A(z)x(z) = H(z)u(z) + \varphi(z), \\ W_2(z)u(z) = W_1(z)x(z), \\ K_\varphi(z) = \sum_{i=-\infty}^{+\infty} \varepsilon(i)z^i. \end{gathered} \tag{4}$$

Control law

Let us now consider the problem of finding the optimal stabilizing control for discrete systems, the formulation of which was formulated in the introduction. Let

$$x(z) = F_x(z)\varphi(z), \qquad u(z) = F_u(z)\varphi(z), \tag{5}$$

where $F_x(z), F_u(z)$ - the corresponding transfer functions. From the equations (4) we have

$$F_x(z) = \left[A(z) - H(z)W(z)\right]^{-1}, \quad F_u(z) = W(z)\left[A(z) - H(z)W(z)\right]^{-1}. \tag{6}$$

Let us represent criterion (2) in the form [2,3]:

$$I = \frac{1}{2\pi j}\oint_\Gamma \left[\lambda K_x + \mu K_u\right]\frac{dz}{z} = \frac{1}{2\pi j}\oint_\Gamma \left[\lambda K_x K_x^* + \mu K_u K_u^*\right]K_\varphi \frac{dz}{z}, \tag{7}$$

where Γ - a circle of unit radius centered at point zero, K_x and K_u – sequence correlation images $\{x_t\}, \{u_t\}$, associated with the correlation images of the sequence $\{\varphi_t\}$:

$$K_x = F_x F_x^* K_\varphi, \qquad K_u = F_u F_u^* K_\varphi.$$

Above and below, as in the case of discrete-continuous systems (DCS), the symbol «*» we will denote the replacement z on the z^{-1}. Sign «~»will denote the conjugate polynomial. A polynomial in which the order of the coefficients is inverse with respect to the original.

Obviously, transfer functions (6) satisfy the constraint equation

$$AF_x - AF_u = 1. \tag{8}$$

We introduce a variable function, similarly to [3,7], in this form:

$$\Phi = \alpha F_x + \beta F_u, \tag{9}$$

where α and β - polynomials such that all zeros of the function Φ lie outside the circle of unit radius. Then from equations (8), (9) we can express K_x and K_u through the function Φ:

$$F_x = \frac{\Phi H + \beta}{\beta A + \alpha H}, \quad F_u = \frac{\Phi H - \alpha}{\beta A + \alpha H}. \tag{10}$$

Let,

$$Q = \beta A + \alpha H \tag{11}$$

is the polynomial in the denominator of expressions (10). Then, taking into account (10) and (11), based on (7), the function turns in this form:

$$\delta I = \frac{1}{2\pi j}\oint_{\Gamma}\left[\lambda(\Phi H+\beta)(\Phi^{*}H^{*}+\beta^{*})+\mu(\Phi A-\alpha)(\Phi^{*}A^{*}-\alpha^{*}\right]Q^{-1}(Q^{*})^{-1}K_{\varphi}\frac{dz}{z}=$$

$$=\frac{1}{2\pi j}\oint_{\Gamma}\left[(\lambda HH^{*}+\mu AA^{*})\Phi\,\Phi^{*}+(\lambda H\beta^{*}-\mu A\alpha^{*})\Phi+\right.$$

$$\left.+(\lambda H^{*}\beta+\mu A^{*}\alpha)\Phi^{*}+(\lambda\beta\beta^{*}-\mu\alpha\alpha^{*}\right]Q^{-1}(Q^{*})^{-1}K_{\varphi}\frac{dz}{z}.$$

Find the function G as a result of factoring the expression:

$$\lambda HH^{*}+\mu AA^{*}=GG^{*}. \tag{12}$$

Let,

$$\lambda\beta H^{*}+\mu A^{*}\alpha=L. \tag{13}$$

Then, taking into account (12), (13), the first variation of the function can be rewritten as:

$$\delta I=\frac{1}{2\pi j}\oint_{\Gamma}\left[\frac{GG^{*}NN^{*}}{QQ^{*}DD^{*}}\Phi+\frac{LNN^{*}}{QQ^{*}DD^{*}}\right]\delta\Phi\frac{dz}{z}=\frac{1}{2\pi j}\oint_{\Gamma}\left[\frac{GG^{*}NN^{*}}{QQ^{*}DD^{*}}\Phi+\frac{L^{*}NN^{*}}{QQ^{*}DD^{*}}\right]\delta\Phi\frac{dz}{z}, \tag{14}$$

where the functions N, D are obtained as a result of factorizing the fraction:

$$K_{\varphi}(z)=\frac{NN^{*}}{DD^{*}}. \tag{15}$$

We will use the vital condition for an extremum, i.e. condition $\delta I = 0$. Note that if function Φ is chosen, the first of the integrals in expression (14) is equal to zero, then automatically, due to symmetry, the second integral in (14) will also be equal to zero. Therefore, in order to find Φ, we will consider only the first integral:

$$\frac{1}{2\pi j}\oint_{\Gamma}\left[\frac{GG^{*}NN^{*}}{QQ^{*}DD^{*}}\Phi+\frac{LNN^{*}}{QQ^{*}DD^{*}}\right]\delta\Phi^{*}\frac{dz}{z}=\frac{1}{2\pi j}\oint_{\Gamma}\frac{G^{*}N^{*}}{Q^{*}D^{*}}\left[\frac{GN}{QD}\Phi+\frac{LN}{QDG^{*}}\right]\delta\Phi^{*}\frac{dz}{z}. \tag{16}$$

Let the fraction,

$$\frac{LN^{*}}{QDG^{*}}=L^{+}+L^{-}, \tag{17}$$

where L^{-} – regular fraction with poles in the area $Z^{-}=\{z\,\|z|\le 1\}$, but L^{+} – fraction with poles outside the unit circle.

Then integral (16) can be represented as a sum of two integrals

$$\frac{1}{2\pi j}\oint_{\Gamma}\frac{G^{*}N^{*}}{Q^{*}D^{*}}\left[\frac{GN}{QD}\Phi+\frac{LN}{QDG^{*}}\right]\delta\Phi\frac{dz}{z}=\frac{1}{2\pi j}\oint_{\Gamma}\frac{G^{*}N^{*}}{Q^{*}D^{*}}\left[\frac{GN}{QD}\Phi+L^{+}\right]\delta\Phi^{*}\frac{dz}{z}+\frac{1}{2\pi j}\oint_{\Gamma}\frac{G^{*}N^{*}}{Q^{*}D^{*}}L^{-}\delta\Phi^{*}\frac{dz}{z}.$$

Since, the integral has poles only inside the unit circle in the second term, then:

$$\frac{1}{2\pi j}\oint_{\Gamma}\frac{G^{*}N^{*}}{Q^{*}D^{*}}L^{-}\delta\Phi^{*}\frac{dz}{z}=0.$$

It follows that the function Φ, which makes the first variation of function (14) zero, is found by the formula:

$$\Phi=-\frac{QD}{GN}L^{+}. \tag{18}$$

Then, due to the relations (6), (7), the transfer function of the optimal controller is equal to

$$W=\frac{\Phi A-\alpha}{\beta+\Phi H}. \tag{19}$$

After substitution of the corresponding functions in (19), we obtain the expression:

$$W(z)=\frac{W_1(z)}{W_2(z)}=\frac{-\lambda(G^*)^{-1}H^*N+ADL^-}{\mu(G^*)^{-1}A^*N+HDL^-}. \tag{20}$$

Based on the above relations, it is easy to obtain the minimum value of the control quality criterion (7) for system (1) closed by controller (3) with transfer function (20):

$$I_{opt}=\frac{1}{2\pi j}\oint_{\Gamma}\left[\frac{\lambda\mu K_\varphi}{GG^*}+L^-(L^-)^*\right]\frac{dz}{z}. \tag{21}$$

Note that, as in the case of continuous and discrete-continuous systems, the transfer function does not depend on the choice of arbitrary polynomials α and β. The only restriction on polynomials α and β is that all zeros of polynomial Q lie outside the unit circle.

Solution

Similarly to how it was done for continuous and discrete-continuous systems, we modify the algorithm for constructing the transfer function of the optimal controller in order to simplify the computational scheme for the synthesis of obtaining polynomials W_1 and W_2 of the indicated transfer functions, by passing the choice of α and β, on which, as noted, does not depend the final result.

Let $G(z)=m<k+l$. Then, it is easy to see, the correct fraction L^- in relation (17) is determined by the zeros of the polynomial $\tilde{G}(z)=z^mG(z^{-1})$ и $k+l+m$ – multiple zero at point $z=0$. Thus, relation (17) can be represented in the form [3]:

$$\frac{(\lambda\beta N^*-\mu\alpha A^*)N}{QDG^*}=L^++\frac{R}{z^{k+l-m}\tilde{G}}, \tag{22}$$

where R – some polynomial.

Let us obtain formulas for constructing polynomial R depending on the values of parameters λ and μ.

Let $\lambda\neq 0$, then the left-hand side of relation (22) can be represented as:

$$\frac{(\lambda\beta N^*-\mu\alpha A^*)N}{QDG^*}=\frac{(\lambda\beta N^*-\mu\alpha A^*)NA}{QDG^*A}=$$
$$=\frac{\left(\lambda AH^*\beta-\alpha\left[GG^*-\lambda HH^*\right]\right)N}{QDG^*A}=\frac{\left(\lambda H^*Q-\alpha GG^*\right)N}{QDG^*A}.$$

The founded values make it possible to construct a polynomial according to the well-known formulas for the expansion of a fraction into simplest ones [7-10]:

$$R(z)=\left[\sum_{j=1}^{m}\frac{\lambda\tilde{H}_1(g_j)N(g_j^{-1})g_j^{k+l-m}}{A(g_j^{-1})D(g_j^{-1})\tilde{G}'(g_j^{-1})z-g_j^{-1}}\frac{1}{z-g_j^{-1}}+\sum_{j=1}^{k+l-m}\frac{1}{(i-1)!}\frac{d^{i-1}}{dz^{i-1}}\left[\frac{\lambda\tilde{H}_1N}{AD\tilde{G}}\right]_{z=0}z^{m-k-l+i-1}\right]\tilde{G}z^{k+l-m}=$$
$$=\left[\sum_{j=1}^{m}\frac{\lambda H_1(g_j)N(g_j^{-1})}{g_j^mA(g_j^{-1})D(g_j^{-1})\tilde{G}'(g_j^{-1})}\cdot\frac{1}{z-g_j^{-1}}+\sum_{j=1}^{k+l-m}\frac{1}{(i-1)!}\frac{d^{i-1}}{dz^{i-1}}\left[\frac{\lambda\tilde{H}_1N}{AD\tilde{G}}\right]_{z=0}z^{m-k-l+i-1}\right]\tilde{G}z^{k+l-m}. \tag{23}$$

Now let $\mu\neq 0$. Then we will write the left-hand side of relation (22) in the form

$$\frac{(\lambda\beta H^*-\mu\alpha A^*)N}{QDG^{-1}}=\frac{(\lambda\beta H^*-\mu\alpha A^*)NH}{QDG^{-1}H}=$$
$$=\frac{(\beta\left[GG^*-\mu AA^*\right]-\mu\alpha A^*H)N}{QDG^{-1}H}=\frac{\left(\beta GG^*-\mu A^*Q\right)N}{QDG^*H}. \tag{24}$$

Taking into account the performed transformations from relation (22), we express $R(z)$, we obtain:

$$R(z)=\frac{\left(\beta GG^{*}-\mu A^{*}Q\right)N}{QDG^{*}H}\tilde{G}z^{k+l-m}-L^{+}\tilde{G}z^{k+l-m}=\frac{\beta G\tilde{G}N}{QDH}z^{k+l-m}-\frac{\mu\tilde{A}N}{DH}z^{k+l-m}-L^{+}\tilde{G}z^{k+l-m}. \qquad (25)$$

The resulting expression allows us to find the value of polynomial $R(z)$ at the points coinciding with the poles of the fraction L^{-}. In addition, the desired polynomial $R(z)$ in this case will have a $(k+l-m)$-fold zero at point $z=0$, since for $\mu\neq 0$, obviously, $m\geq\delta_A=\deg A(z)$.

Therefore:

$$R(g_i^{-1})=\frac{\mu A(g_i)N(g_i^{-1})}{g_i^{k+l}D(g_i^{-1})H(g_i^{-1})},R(0),R'(0)=0,\ldots,R^{k+l-m-l}(0)=0,$$

a, therefore, we finally have:

$$\begin{aligned}R(g_i^{-1})&=-\sum_{i=1}^{m}\frac{\tilde{G}}{z-g_i^{-1}}\cdot\frac{\mu A(g_i)N(g_i^{-1})g_i^{k+l-m}}{g_i^{k+l}D(g_i^{-1})H(g_i^{-1})\tilde{G}'(g_i^{-1})}z^{k+l-m}=\\&=-\sum_{i=1}^{m}\frac{\tilde{G}}{z-g_i^{-1}}\cdot\frac{\mu A(g_i)N(g_i^{-1})}{g_i^{m}D(g_i^{-1})H(g_i^{-1})\tilde{G}'(g_i^{-1})}z^{k+l-m}=R_1z^{k+l-m}.\end{aligned} \qquad (26)$$

Based on (17) and (20), the transfer function of the optimal controller [3,11-15]:

$$\begin{aligned}W(z)&=\frac{W_1(z)}{W_2(z)}=\frac{-\lambda(G^{*})^{-1}H^{*}N+ADL^{-}}{\mu(G^{*})^{-1}A^{*}N+HDL^{-}}=\\&=\frac{(-\lambda z^{m}H^{*}N(G^{*})^{-1}+ADR(G^{*})^{-1})z^{m-k-l}}{(\mu z^{m}A^{*}N(G^{*})^{-1}+HDR(G^{*})^{-1})z^{m-k-l}}=\frac{(-\lambda z^{k+l}H^{*}N+ADR)(G^{*})^{-1}z^{m-k-l}}{(\mu z^{k+l}A^{*}N+HDR)(G^{*})^{-1})z^{m-k-l}},\end{aligned} \qquad (27)$$

Moreover, division by polynomial $\tilde{G}$ in the numerator and denominator is carried out entirely, and polynomial $R(z)$ is constructed by formula (23), or (26).

Let us now consider the case when $m\geq k+I$. Note that the regular fraction L^{-} in relation (17) is completely determined by the zeros of the polynomial $\tilde{G}$, therefore, we can write

$$\frac{(\lambda\beta N^{*}-\mu\alpha A^{*})N}{QDG^{-1}}=L^{+}+\frac{R}{\tilde{G}}. \qquad (28)$$

Transforming the left-hand side of relation (28) and assuming that $\lambda\neq 0$, in accordance with transformation (24), we have [3,11]:

$$R(z)=\lambda z^{m}H^{*}NA^{-1}D^{-1}-\alpha NG\tilde{G}A^{-1}Q^{-1}D^{-1}-L^{+}\tilde{G}.$$

Hence,

$$R(g_i^{-1})=\frac{\lambda H_1(g_i)N(g_i^{-1})}{g_i^{m}D(g_i^{-1})A(g_i^{-1})},\quad i=1,2,\ldots,m,$$

and therefore, finally:

$$R(z)=\sum_{i=1}^{m}\frac{\tilde{G}}{z-g_i^{-1}}\cdot\frac{\lambda H(g_i)N(g_i^{-1})}{g_i^{m}D(g_i^{-1})A(g_i^{-1})\tilde{G}'(g_i^{-1})}. \qquad (29)$$

Accordingly, if $\mu\neq 0$, transformations (24) allow us to write:

$$R(z)=-\mu z^{m}A^{*}NH^{-1}D^{-1}+\beta NG\tilde{G}H^{-1}Q^{-1}D^{-1}-L^{+}\tilde{G}.$$

Its meaning is that,

$$R(g_i^{-1}) = \frac{\mu A(g_i)N(g_i^{-1})}{g_i^m D(g_i^{-1})H(g_i^{-1})}, \quad i = 1, 2, \ldots, m,$$

and therefore in this case:

$$R(z) = -\sum_{i=1}^{m} \frac{\tilde{G}}{z - g_i^{-1}} \cdot \frac{\mu A(g_i)N(g_i^{-1})}{g_i^m D(g_i^{-1})H(g_i^{-1})\tilde{G}'(g_i^{-1})}. \tag{30}$$

We obtain the final formulas for constructing the transfer function of the optimal controller:

$$W_1(z) = \frac{-\lambda z^m H^* N + ADR}{G^*}, \tag{31}$$

$$W_2(z) = \frac{\mu z^m A^* N + HDR}{G^*}, \tag{32}$$

where $R(z)$ is constructed according to formulas either (29) or (30), and division by polynomial $\tilde{G}$ is carried out entirely.

The minimum value of the quality function according to (21), (17), (28) can be calculated by the formula:

$$I_{opt} = \frac{1}{2\pi j}\oint_{\Gamma}\left[\frac{\lambda\mu K_{\varphi}}{GG^*} + L^{-}(L^{-})^*\right]\frac{dz}{z} = \frac{1}{2\pi j}\oint_{\Gamma}\left[\frac{\lambda\mu K_{\varphi} + RR^*}{GG^*}\right]\frac{dz}{z}.$$

Conclusion

In conclusion, an improved method is proposed for constructing optimal controllers that provide a minimum of the root-mean-square performance criterion for discrete-continuous systems, which does not require a laborious auxiliary procedure for solving a system of polynomial equations as one of the stages of constructing a controller.

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