SOME PROPERTIES OF THE DISTANCE BETWEEN TWO POINTS

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In order to teach students to think abstractly, it is advisable to generalize some properties that are appropriate in a straight line or plane.

In this article, we have tried to achieve the goal by introducing the metric space concept studied by distinguishing characteristic properties that have a distance between two points. In particular, we believe that the examples given in this article will further increase the reader's interest and allow a deeper study of the topic.

It is known that the distance between two points can be determined in different ways. For example, the distance between Paris and Rome cities can be determined by air, road or water. In general, if we define the distance between points x and y as $\rho(x,y)$, this two-variable function has the following properties:

1) $\rho(x,y)\geq 0$ ва $\rho(x,y)=0 \Leftrightarrow x=y$
2) $\rho(x,y)=\rho(y,x)$
3) $\rho(x,y)\leq \rho(x,z)+\rho(z,y)$

When variables such as x, y are elements of a definite set X, the function $\rho(x,y)$ is called a metric in that set X, and a set X is called a metric space relative to the set ρ. The distance between any two points in a set, i.e., the metric, allows us to determine many concepts in that set.

For example, concepts such as a sequence convergence consisting of set X elements, try point, limit point, open and closed spheres, open and closed sets, continuous reflection, abbreviated reflections can be introduced. Using the abbreviated reflection concept, it is possible to determine whether some equations have a solution.

Below we consider some examples related to metrics that satisfy the above three conditions.

A set of form

$$S(x_0,r)=\{x\in X \mid \rho(x_0,x)<r\}$$

defined by the metric $\rho(x,y)$ given in set X is called an open sphere in set X. In this case, the point x_0 is called the center of the sphere, and r is called its radius.

If all real numbers in the set are defined by the simple metric $\rho(x,y)=|x-y|$, then the open spheres in it are in the form of intervals. If we determine the metric between the points $x=(x_1,x_2)$ and $y=(y_1,y_2)$ in the set of points in the plane R^2 by the formula

$$\rho(x,y)=\sqrt{(x_1-y_1)^2+(x_2-y_2)^2}$$

then the open spheres in it will consist of all the interior points of the circle.

Taking into account that different metrics can be defined in a single set, the following issues can be addressed.

1. In a metric space, can a sphere with a large radius lie inside a sphere $S(b,r_1)$ (i.e., $r_1 < r$) with a small radius $S(a,r)$?

Let $\rho(x,y)$ be the metric in set X. If $\{x_n\} \subset X$, and the sequence of numbers $\rho(x_n,x)$ tends to 0 at $n \to \infty$, then the sequence $\{x_n\}$ is called convergent, and in this case is denoted by $\lim_{n\to\infty} x_n = x$.

Since it is possible to identify multiple metrics in a set, the given sequence may be approximating to one metric and not approximating to another metric. For example, relatively to discrete metrics, only sequences that are stationary will be approximating.

Indeed, for the sequence $\{x_n\}$ to be close to X, for any number $\varepsilon > 0$ such a number n_0 is found, the inequality $\rho(x_n,x) < \varepsilon$ must be satisfied for all n satisfying the inequality $n_0 \le n$. If $\varepsilon = \frac{1}{2}$, then for all x_n satisfying the inequality $n_0 \le n$ from the property of the discrete metric, the equations $x_n = x_{n+1} = x_{n+2} = ...$ must be satisfied, i.e., the sequence $\{x_n\}$ must be stationary.

We know that, the set of rational numbers Q with respect to a simple metric $\rho(x,y) = |x-y|$ is the metric space in which the sequence $\{x_n\}$ defined by the equation $x_n = \frac{1}{3^n}$ approaches 0. The $\{y_n\}$ sequence defined by the $y_n = 3^n$ equation does not converge. Below we define a metric in set Q such that the $\{y_n\}$ sequence becomes approximate without the $\{x_n\}$ sequence convergent.

2. Let Q be a set of rational numbers and ρ be a prime number. Let Q be a rational number $0 < Q < 1$ satisfy the inequality. Any rational number α can only be expressed in the form $\alpha = \rho^n \frac{a}{b}$ in the same form, where $(a,\rho) = 1$, $(b,\rho) = 1$, and $(a,b) = 1$. In that case, the distance between the rational numbers α and β can be determined in the form

$$\rho(\alpha,\beta) = \begin{cases} Q^n, & \alpha \ne \beta \\ 0, & \alpha = \beta \end{cases}.$$

Here we considered $\gamma = \alpha - \beta = \rho^n \frac{a}{b}$ equations appropriate.

The $\rho(\alpha,\beta)$ functions included can show metric satisfaction in set Q.

References.

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