MATHEMATICAL MODEL OF DYNAMICS OF A DRIVE WITH A PLANETARY MECHANISM

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Abstract.

The possible number of satellites is given in the article; the pitch radii of the wheels for a given modulus are defined; the moment of inertia of the mechanism reduced to the movable central wheel is determined; a mathematical model of the motion of a drive with a planetary gear mechanism is developed. Equations of motion of a drive with a planetary gear mechanism were obtained. Assuming, in a particular case, all the links of the drive with the planetary gear mechanism as rigid links, a mathematical model was developed for this system, considering the dynamic characteristics of an electric motor. Analytical solutions for the developed mathematical model are given.

Keywords:

Drive, planetary gear, number of gear teeth, satellite.

Introduction.

Drives with planetary gear mechanisms are widely used in various branches of technology and industry. Numerous publications [1-4] are devoted to the study and analysis of various mechanisms of various functions. Analyzing numerous studies aimed at improving the operation of the mechanisms and their wide application, the theoretical foundations and designs of planetary and biplanetary drives of the working bodies of doughing machines were developed in [2]. Mathematical models for controlling the parameters and constraints of the friction mechanism were developed in [3]. A review of the development and state of the kinematics and dynamics of the cam mechanism was conducted [4]. Below, the equations of motion of the grinder drive with a planetary gear mechanism are proposed.

Methods.

Research is based on the method of determining the number of gear teeth; the method of determining the kinetic energy of the James gearbox.

Materials.

Determination of the actual laws of motion of the working body of a grinder is a necessary factor in its design. The working body of the grinder receives motion from an asynchronous electric motor through a planetary gearbox.

Consider the methods for determining the number of teeth of a given gearbox [1]. Suppose that it is necessary to design a planetary gear the number of teeth of wheel 3 is

$$z_2 = \frac{z_3 - z_1}{2} \quad (1)$$

or

$$z_2 = \frac{b \cdot z_1 - z_1}{2} = \frac{z_1(b-1)}{2} \quad (2)$$

Considering this, we obtain

$$\frac{z_3}{z_2} = \frac{2 \cdot b}{(b-1)} \quad (3)$$

The number of teeth should be

$$z_2 = \frac{z_1^2 - 34}{34 - 2 \cdot z_1} \quad (4)$$

and for internal gearing it is

$$z_3 \geq \frac{z_2^2 - 34}{2 \cdot z_2 - 34} \quad (5)$$

Now let us determine the possible number of satellites K using the conditions of neighborhood and assembly

$$K < \frac{\pi}{a \sin \frac{z_2 + 2}{z_1 + z_2}}, \qquad K = \frac{z_1 + z_3}{n} \quad (6)$$

Next, we determine the pitch radii of the wheels for a given modulus m

$$z_1 = \frac{m \cdot z_1}{2}, \; z_2 = \frac{m \cdot z_2}{2}, \; z_3 = \frac{m \cdot z_3}{2}$$

For values of $u_{1H}^{(3)}$ equal to 4, 5, 6, 7, 8, the number of gears teeth 1, 2, 3, the number of possible satellites, and the pitch radii of these gears were determined. The results are shown in table 1.

Table 1. Values of geometrical parameters of the James gearbox.

№	u_{1H}^3	u_{H1}^3	m	z_1	z_2	z_3	K	R_1	R_2	R_3
1	4	0,25	5	20	20	60	5	50	50	150
2	5	0,2	5	18	25	68	2	10	62,5	170
3	6	0,17	5	17	34	85	3	42,5	85	212,5
4	7	0,143	5	18	45	108	3	45	112,5	270
5	8	0,125	5	18	50	120	3	45	125	300

We define the kinetic energy of the James gearbox as:

$$T = T_g + T_3 + T_c + T_H$$

where T_g – is the kinetic energy of the electric motor rotor; T_3 – is the kinetic energy of the gear 3; T_c – is the kinetic energy of satellites; T_H – is the kinetic energy of the carrier. Or

$$T = \frac{1}{2}\left(J_g \cdot \omega^2 + J_3 \cdot \omega_3^2 + k \cdot J_2 \cdot \omega_2^2 + K \cdot m_2 \cdot R_H^2 \cdot \omega_H^2 + J_H \cdot \omega_H^2\right)$$

where J_g – is the moment of inertia of the electric motor rotor; J_3 – is the moment of inertia of gear 3 relative to the axis of rotation; J_2 – is the moment of inertia of the satellite relative to its axis of rotation; $K-$ is the satellite number, R_H – is the radius of the satellite center trajectory; m_2- is the mass of the satellite, J_H- is the moment of inertia of the carrier relative to the axis of rotation.

Let us determine the moment of inertia of the mechanism rated to wheel 3. In this case, the working disk is connected to the carrier, and wheel 3 is the driving link, then

$$J_{np1} = J_g + J_3 + K \cdot J_2 \left(\frac{\omega_2}{\omega_3}\right)^2 + K \cdot m_2 \cdot R_H^2 \cdot \left(\frac{\omega_H}{\omega_3}\right)^2 + J_H \left(\frac{\omega_H}{\omega_3}\right)^2 + J_P \left(\frac{\omega_H}{\omega_3}\right)^2$$

The mathematical model for this system, considering the dynamic characteristics of the electric motor, has a well-known form [1]:

$$\omega_\partial = \omega_0 \left[1 - \nu_\partial \left(M_\partial + T_\partial \cdot \frac{dM_\partial}{dt}\right)\right]$$
$$\frac{d\omega_\partial}{dt} = \frac{1}{I_\Pi}(M_\partial - M_c) \qquad (7)$$

where ω_∂ – is the angular speed of the electric motor rotor and the disk on which the rotating working bodies are installed; I_Π – is the total moment of inertia of the electric motor rotor and the disk with the working bodies; M_∂ – is the moment developed by the asynchronous electric motor; M_c – is the resulting moment from the resistance forces acting on the working body; ω_∂ is the angular speed of the rotor of the electric motor and the disk with working bodies; ω_0 – is the angular speed of the ideal no-load speed of the electric motor; ν_∂ – is the steepness of the static characteristic; M_∂ – is the moment developed by the electric motor; T_∂ – is the electromagnetic time constant of the engine, depending on the parameters of its electrical circuit.

For an asynchronous electric motor with a squirrel-cage rotor, ν_∂ and T_∂ can be determined by the following dependencies:

$$T_\partial = \frac{1}{2\pi f_c S_\kappa}; \qquad \nu_\partial = \frac{S_\kappa}{2M_\kappa \cdot \lambda}; \qquad S_\kappa = \left(1 - \frac{\omega_H}{\omega_\partial}\right)\left(\lambda + \sqrt{\lambda^2 - 1}\right)$$

where M_H – is the nominal moment of the motor; λ – is the ratio of the critical moment to the nominal moment (overload factor); f_c – is the circuit frequency; S_κ – is the critical slip; ω_H – is the motor rated angular speed.

Taking into account the above, the equation of motion of the grinder drive can be written in the following form:

$$\nu_\partial T_\partial I_\Pi \ddot{\omega}_\partial + \nu_\partial I_\Pi \dot{\omega}_\partial + \frac{\omega_\partial}{\omega_0} = 1 - \nu_\partial \left(M_c + T_\partial \cdot \dot{M}_c\right) \qquad (8)$$

we assume that $T_\partial = 0$, then

$$\omega_\partial^* = \omega_0 \left(1 - \nu_\partial \cdot M_c^*\right) \qquad (9)$$

Taking into account expression (9), equation (8) can be written in the following form:

$$\omega_\partial^* + 2n\dot{\tilde{\omega}}_\partial + \kappa^2 \tilde{\omega}_\partial = W(t) \qquad (10)$$

where $n = \frac{1}{2T_\partial}$; $\kappa^2 = \frac{1}{\nu_\partial \cdot T_\partial \cdot I_\Pi \cdot \omega_0}$; $W(t) = -\frac{1}{I_\Pi}\left(\frac{M_c}{\tilde{T}_\partial} + \dot{\tilde{M}}_c\right)$

To solve equation (6), we use the Fourier series [2]. For the case of a harmonic exciting force, the expression for forced vibrations is written in the following form

$$q = A_0 + \sum_{j=1}^{\infty} A_j \sin\left(j\omega t + \theta_j - \Delta_j\right)$$

where $A_0 = Q_0/c$; $A_j = A_{jcm} K_g$; $A_{jcm} = Q_j/c$.

In the grinder of mineral raw materials, considered in the article, an asynchronous electric motor is used with a power of N=18,5kW; rated speed of n_H = 3000rpm. Let us determine the angular velocity of the electric motor ω_∂.

$$J_ï = 0{,}1\,kg \cdot m^2;$$

$$M_C = \left(50 - 18{,}2\cos\omega\cdot t - 15\sin\omega\cdot t + 9\cos 2\omega\cdot t - 20\sin 2\omega\cdot t\right) N\cdot m \text{ at } \omega = 90 s^{-1}.$$

Determine nominal moment $Ì_í = 9549\dfrac{N}{n_H} = 60{,}1\,N\cdot m$.

Nominal slip is

$$S_H = 1 - \frac{n_H}{n_0} = 0{,}02; \qquad S_k \approx S_H\left(\lambda + \sqrt{\lambda^2 - 1}\right) = 0{,}057;$$

$$\nu = \frac{S_k}{2M_H \cdot \lambda} = 2{,}96\cdot 10^{-4}\frac{1}{N\cdot m}. \qquad T_g = \frac{1}{2\pi\cdot f_c \cdot S_k} = 0{,}056\ s,$$

$$\omega_g^* = \omega_0\left(1 - \nu_g M_c^*\right) = 309{,}5\ s^{-1},$$

here $\omega_0 = \dfrac{\pi n_0}{30} = 314{,}2\ s^{-1}$; $M_c^* = 50\ N\cdot m\ s^{-3}$,

$$W = Q_1\sin(\omega t + \theta_1) + Q_2\sin(2\omega t + \theta_2),$$

where $Q_1 = 40000\ s^{-3}$; $Q_2 = 38000\ s^{-3}$; $\theta_1 = 89^\circ$; $\theta_2 = 32^\circ$.

The solution to the differential equation (10) is

$$\omega_g = \omega_1\sin(\omega t + \theta_1 - \Delta_1) + \omega_2\sin(2\omega t + \theta_2 - \Delta_2)$$

where $\omega_1 = \dfrac{Q_1}{\sqrt{\left(k^2 - \omega^2\right)^2 + 4n^2\omega^2}}$; $\omega_2 = \dfrac{Q_2}{\sqrt{\left(k^2 - 4\omega^2\right)^2 + 16n^2\omega^2}}$;

$$\Delta_1 = arctg\frac{2n\omega}{k^2 - \omega^2}; \Delta_2 = arctg\frac{4n\omega}{k^2 - 4\omega^2};$$

$$k = \frac{1}{\nu_g T_g J_n \omega_0} = 43{,}85 s^{-1}; n = \frac{1}{2T_g} = 8{,}95 s^{-1};$$

Results and discussion.

On the basis of the analytical expressions obtained, the pattern of changes in the angular velocity of the driving link was determined for the steady-state operation of the grinder. The calculations showed that with the above parameters, the peak-to-peak amplitude of the angular velocity of the electric motor rotor is approximately 18.6 s $^{-1}$.

Based on the study conducted, the following results (conclusions) were drawn:

1. A mathematical model of the motion of a drive with a planetary gear mechanism was developed.
2. On the basis of the analytical expressions obtained with the developed mathematical model, the pattern of changes in the angular velocity of the driving link in the steady-state operation of the grinder was determined.

References

1. Artobolevsky I.I. The theory of mechanisms and machines. - M.: Nauka, 1988. -640 p.
2. Karimov R.I. Theoretical foundations and designs of planetary and biplanetary drives of working bodies of doughing machines: Monograph. –Tashkent: Tashkent State Technical University, 2013. – 145 p.
3. Yang, Yuhu. Design of a novel coaxial eccentric indexing cam mechanism. Mechanism and

Machine Theory, vol.132, February 2019, P. 1-12.
4. https://doi.org/10.1016/j.mechmachtheory.2018.10.012
5. Fan Y Chen. A survey of the state of the art of cam system dynamics. Mechanism and Machine Theory, vol.12, issue 3, 1977, P. 201-224.
6. https://doi.org/10.1016/0094-114X(77)90020-9