MODELING THE PROCESS OF FORCE LOAD GENERATION AT PERIODIC CHANGE IN PRESSURE

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ABSTRACT.

An analytical solution to the problem of pulsating fluid motion in a plane-parallel channel is obtained with allowance for single and group transfer of molecules in the flow. The application of the analytical expressions obtained for the velocities is not limited to the critical Reynolds number, i.e. they are applied for any values of this number. The resulting solution describes two zones of flow: in the first zone, two types of transfer occur, depending on the flow pattern, either molecular or molar transfer of fluid volumes between the layers prevails. In the second zone, only molecular transfer occurs.

KEY WORDS:

Fluid flow, molecular and molar transfer in a flow, pulsating flows, fluid velocity, Navier-Stokes equation

Introduction.

Pulsating flows of fluid ensure the existence of biological and social objects and they are an integral part of the technological production processes. The flow in a sudden expansion channel and its flow through an idealized curved coronary artery with a pulsating velocity at the inlet was studied in [1]. A pulsating flow for thermohydraulic analysis of a nuclear reactor in an oceanic environment was investigated in [2]. Modeling a pulsating inlet flow to study the performance of flutterbased energy harvesters [3] and the effect of pseudo-plastic fluid flow in a manifold microchannel heat sink [4] is the evidence of the widespread use of pulsating flow.

In this study, to account for the transfer of a substance between layers at the molecular level, it is assumed that the stress is directly proportional to the derivative of the normal velocity, and with all the above circumstances, during molar transfer, the stress is proportional to the derivative of the normal acceleration [5].

Methods.

Research methods are based on the method of mathematical modeling and the analytical methods for their solution, based on the provisions of operational calculus.

Materials.

Let us consider a plane-parallel pulsating flow of fluid, taking into account the molecular and molar transfer in the flow. The system of equations of fluid motion, in this case, has the following form [4]:

$$\left.\begin{aligned} &\rho\frac{\partial v_1}{\partial t} = -\frac{\partial P}{\partial x} + \mu\frac{\partial^2 v}{\partial y^2} + m_l\left(\frac{\partial^3 v_1}{\partial t\partial y^2} + v_1\frac{\partial^3 v_1}{\partial x\partial y^2}\right),\\ &\frac{\partial v_1}{\partial x} + \frac{\partial v_2}{\partial y} = 0 \end{aligned}\right\} \qquad (1)$$

Where x,y –are the coordinates; t is time; V_1, V_2 are the velocity components; P is the pressure; μ is the dynamic viscosity; m_e is the molar transfer coefficient.

The pressure gradient is given as:

$$-\frac{\partial P}{\partial x} = a + b\cos\omega t \qquad (2)$$

The initial and boundary conditions are as follows:

$$\begin{aligned} &v_1(x,y,0) = \frac{\partial v_1}{\partial y}_{|t=0} = \frac{\partial^2 v_1}{\partial y^2}_{|t=0} = 0,\\ &v_1(0,y,t) = u_0\left(1-e^{-\gamma t}\right); \frac{\partial v_1}{\partial y}_{|x=0} = \frac{\partial^2 v_1}{\partial y^2}_{|x=0} = 0,\\ &\frac{\partial v_1}{\partial y}_{|y=0} = v_2(x,0,t) = 0,\\ &v_1(x,L,t) = v_2(x,L,t) = 0,\\ &\lim_{x\to\infty} v_1(x,y,t) = M = const. \end{aligned} \qquad (3)$$

here γ is a parameter that takes into account the instantaneous transition of the velocity at $x = 0$ from the state of rest in terms of velocity u_0=*const*.

In order to obtain an analytical solution to this problem, we introduce the following function:

$$u(x,y,t) = v_1(x,y,t) - u_0(1-e^{-\gamma t}). \qquad (4)$$

We introduce function $\tilde{w}(x,r,p) = \bar{u}(x,r,p) - A(p)$, and apply the Laplace transform in x to the resulting equation, and a second-order differential equation with respect to the function $\tilde{w}$ is obtained:

$$\frac{\partial^2 \tilde{w}}{\partial y^2} = \frac{\rho p}{\mu + m_l p + m_l u_0 s}\tilde{w} = 0\ , \qquad (5)$$

there is a solution:

$$\tilde{w} = -\left[\frac{a}{\mu s p^2} + \frac{b}{\rho s(p^2+\omega^2)}\right]\frac{ch[\xi(p,s)y]}{ch[\xi(p,s)L]}. \qquad (6)$$

Here p, s are the parameters of the Laplace transform in t and x, respectively.

$$\xi^2(p,s) = \frac{\rho p}{\mu + m_l p + m_l u_0 s}. \qquad (7)$$

Using the Cauchy theorem [5], we obtain:

$$\frac{ch[\xi(p,s)y]}{ch[\xi(p,s)L]} = \frac{\pi}{L^2}\sum_{n=1}^{\infty}\frac{(-1)^{n-1}(2n-1)}{\xi^2(p,s) + \frac{\pi^2}{4L^2}(2n-1)^2}\cos\frac{\pi y(2n-1)}{2L} \qquad (8)$$

now (5) is written as:

$$\bar{w}(s,y,p)=-\frac{4}{\pi m_l u_0 s}\left[\frac{a}{\mu p^2}+\frac{b}{\rho\left(p^2+\omega^2\right)}\right]\sum_{n=1}^{\infty}\frac{(-1)^{n-1}\left[\mu+m_l p+m_l u_0 s\right]\cos\frac{\pi y(2n-1)}{2L}}{(2n-1)\left[s+\frac{\mu}{m_l u_0}+p\frac{4\rho L^2+m_l\pi^2(2n-1)^2}{m_l u_0\pi^2(2n-1)^2}\right]} \quad (9)$$

Performing the inverse transformation sequentially with respect to parameters s and p from [5], we obtain:

$$w(s,y,t)=-\frac{4}{\pi\mu}\sum_{n=1}^{\infty}\left\{\psi(t,n)\delta(t)-\exp\left[-\frac{\mu x}{m_l u_0}\right]\psi\left(t-\frac{\mu x}{m_l u_0\delta_n},n\right)\delta\left(\frac{m_l u_0\delta_n t}{\mu}-x\right)\right\}\cos\frac{\pi y(2n-1)}{2L}-$$
$$-\frac{4}{\pi\mu}\exp\left[-\frac{\mu x}{m_l u_0}\right]\sum_{n=1}^{\infty}\frac{(-1)^{n-1}\cos\frac{\pi y(2n-1)}{2n-1}}{(2n-1)}\left\{a\left(t-\frac{\mu x}{m_l u_0\delta_n}\right)+\frac{b\mu}{\rho\omega}\sin\omega\left[t-\frac{\mu x}{m_l u_0\delta_n}\right]\right\}\delta\left(\frac{m_l u_0\delta_n t}{\mu}-x\right), \quad (10)$$

where $\delta(t)=\begin{cases}1, t>0\\ 0, t<0\end{cases}$ is the unit Heaviside function.

From (10) it follows that there are two zones in the flow $0\le x\le u_0 m_l\delta_n t/\mu$ and $x>u_0 m_l\delta_n t/\mu$. For the second zone, the solution takes the following form:

$$w^*(x,y,t)=-\frac{4}{\pi\mu}\sum_{n=1}^{\infty}\left\{\psi(t,n)\cos\frac{\pi y(2n-1)}{2L}\right\}. \quad (11)$$

Moving on to the initial functions, we finally determine:

$$v_1(x,y,t)=w(x,y,t)+\frac{at}{\rho}+\frac{b}{\rho\omega}\sin\omega t,$$

$$v_2(x,y,t)=\frac{8L^2}{\pi^2 m_l u_0}\exp\left[-\frac{\mu x}{m_l u_0}\right]\sum_{n=1}^{\infty}\frac{(-1)^{n-1}\sin\frac{\pi y(2n-1)}{2L}}{(2n-1)^2}\times \quad (12)$$
$$\times\left\{a\left(t-\frac{\mu x}{m_l u_0\delta_n}\right)+\frac{b}{\rho}\left[\frac{\mu}{\omega}\sin\omega\left(t-\frac{\mu x}{m_l u_0\delta_n}\right)-m_l\cos\omega\left(t-\frac{\mu x}{m_l u_0\delta_n}\right)\right]\right\}\delta\left(\frac{m_l u_0\delta_n t}{\mu}-x\right)$$

Results and discussion.

An analytical solution to the problem of plane-parallel pulsating flow of fluid is obtained, taking into account the molar transfer in the flow.

The following results (conclusions) were obtained based on the study:

1. The application of the analytical expressions obtained, for the velocities is not limited to the critical Reynolds number, i.e. they are applied for any values of this number, and also describe the annular Richardson effect, which is of great practical importance in reducing hydro-erosion in pipes transporting suspensions, dusty gases and other substances.
2. The solution obtained shows that there are two flow zones: in the first zone there are two types of motion, depending on the nature of the flow, where either molecular or molar transfer of fluid volumes between the layers prevails. In the second zone, only molecular transfer occurs. This means that with time, the flow passes into the limiting mode.

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